

Drell-Yan process at forward rapidity at the LHC

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We analyze the Drell-Yan lepton pair production at forward rapidity at the Large Hadron Collider. Using the dipole framework for the computation of the cross section we find a significant suppression in comparison to the collinear factorization formula due to saturation effects in the dipole cross section. We develop a twist expansion in powers of Q_s^2/M^2 where Q_s is the saturation scale and M the invariant mass of the produced lepton pair. For the nominal LHC energy the leading twist description is sufficient down to masses of 6 GeV. Below that value the higher twist terms give a significant contribution.

I. INTRODUCTION

The Large Hadron Collider (LHC) opens a new kinematic regime at high energies. In this regime QCD evolution leads to the fast growth of the gluon density. At these high densities it is possible that the novel phenomena related to the nonlinear dynamics of the gluon fields will occur. Drell-Yan production is a unique process which offers high sensitivity to the parton distribution in the hadron. It is one of the few processes in hadron-hadron collisions where the collinear factorization has been rigorously proven [1–4]. Within this framework, the NLO calculations have been performed in [5–7], and later on up to NNLO accuracy in [8–10]. In the collinear factorization approach the DY process is viewed as the fusion of the quark and antiquark which produces a virtual (timelike) photon.

One can view the same process alternatively in the rest frame of one of the hadrons. The quark (typically valence one) from the fast incoming projectile interacts with the color field of the target hadron, and emits the virtual photon. The photon then decays producing lepton pair which moves into the region of forward rapidity (with respect to the incoming projectile). The original description of the DY in the dipole picture has been proposed in [11, 12] with details of the calculations presented in [13]. This process has been also reexamined in [14] and later on in [15, 16] within the framework of the Color Glass Condensate. This formulation is applicable to the very forward production, when the fractions of the longitudinal momenta of the incoming partons are very different (see discussion in the next section). This approach has been very useful as one can easily incorporate the higher twist effects due to the multiple scattering of dipole off the target field. When the energy is high, one of the momentum fractions of the incoming partons is very small, and as a result one is probing potentially dense gluon fields in the target. This means that the multiple scatterings have to be taken into account. Consequently the parton evolution should be modified by inclusion of higher twist terms, i.e. terms which are subleading in the expansion of $Q_s(x)^2/M^2$ where $Q_s(x)$ is the saturation scale characterizing the dense gluon field in the target, and M^2 is the invariant mass squared of the produced Drell-Yan pair.

The twist expansion method for analyzing the deep inelastic process has been constructed in [17] for the case of the dipole model based on the idea presented in [18]. There, a systematic expansion in powers of $Q_s(x)^2/Q^2$ was performed and terms up to twist 4 were extracted analytically both for the longitudinal and transverse structure function. The analysis was further extended to the dipole model which included the DGLAP evolution corrections [19].

In this paper we develop a twist expansion for the Drell-Yan process in a similar approach to that presented in [17] for DIS. Unlike the DIS case however, due to the presence of the second hadron (projectile) the expansion involves additional dependence on the structure function of that fast moving hadron. As a result the obtained expressions still contain the integrals over the longitudinal momentum fractions which involve the structure function. Using this formalism we evaluate the twist 2 and twist 4 contributions to the DY process as a function of the DY mass. We find that for the nominal LHC energy 14 TeV, the leading twist 2 contribution coincides with the all-twist result for masses down to about $M = 6$ GeV. Below that value resummation of the higher twists contributions is necessary. We have

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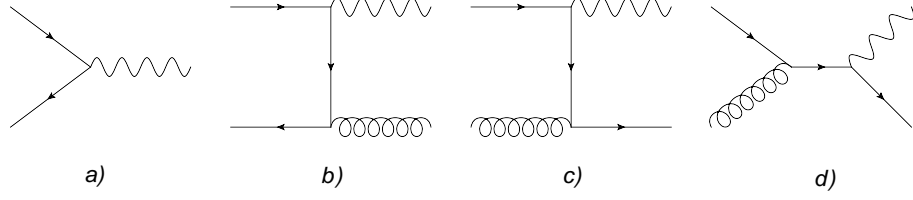


FIG. 1: The leading and next-to-leading order diagrams for the Drell-Yan production. The diagrams c) and d) are enhanced in the small- x limit due to a strongly rising gluon distribution.

performed the analysis for both the transverse and longitudinal components and find that the twist 4 contributions have different signs in both cases (for ver low masses), however we do not observe the cancellations of the type found in the case of the DIS process [17].

II. DRELL-YAN CROSS SECTION

In the lowest approximation, the Drell-Yan lepton pair of mass M is produced form annihilation of two quarks of the same flavour f from the colliding hadrons: $q_f \bar{q}_f \rightarrow \gamma^* \rightarrow l^+ l^-$, see Figure 1a. In the collinear factorization approach, the leading order (LO) Drell-Yan cross section is given by

$$\frac{d^2 \sigma^{LO}}{dM^2 dx_F} = \frac{4\pi\alpha_{em}^2}{3N_c M^4} \frac{x_1 x_2}{x_1 + x_2} \sum_f e_f^2 \{q_f(x_1, M^2) \bar{q}_f(x_2, M^2) + \bar{q}_f(x_1, M^2) q_f(x_2, M^2)\}, \quad (1)$$

where α_{em} is the fine structure coupling constant, $N_c = 3$ is the number of quark colors, q_f/\bar{q}_f are quark/antiquark distributions in the colliding hadrons computed at the factorization scale $\mu^2 = M^2$ and $x_{1,2}$ are the light-cone momentum fractions of the quarks entering the scattering. In the LO approximation, the energy-momentum conservation at the photon vertex, $(x_1 p + x_2 \bar{p})^2 = M^2$, leads to the following relation

$$x_1 x_2 = M^2/s \equiv \tau, \quad (2)$$

where $s = (p + \bar{p})^2 = 2p \cdot \bar{p}$ is the center-of-mass energy squared of the colliding hadrons. Introducing the Feynman variable of the lepton pair, $x_F = x_1 - x_2$, one can easily find

$$x_1 = \frac{1}{2}(\sqrt{x_F^2 + 4\tau} + x_F), \quad x_2 = \frac{1}{2}(\sqrt{x_F^2 + 4\tau} - x_F). \quad (3)$$

In the next-to-leading order (NLO) approximation, additional emission of a parton (quark or gluon) into the final state has to be taken into account. This is shown by the diagrams in Figure 1b-1d. Because of the emission, the quark entering the photon vertex carries a fraction $z < 1$ of the original parton momentum. Thus, the energy-momentum conservation at the photon vertex, e.g. $(x_1 p + z x_2 \bar{p}) = M^2$, gives now

$$x_1 x_2 = \tau/z, \quad (4)$$

and the parton momentum fractions take the form

$$x_1 = \frac{1}{2}(\sqrt{x_F^2 + 4(\tau/z)} + x_F), \quad x_2 = \frac{1}{2}(\sqrt{x_F^2 + 4(\tau/z)} - x_F). \quad (5)$$

From the parton model conditions, $x_{1,2} < 1$, we find that $z > z_{min} = \tau/(1 - x_F)$. The NLO correction to the Drell-Yan cross section, proportional to the strong coupling constant α_s , is given in the $\overline{\text{MS}}$ factorization scheme by [5–7]

$$\begin{aligned} \frac{d^2 \sigma^{NLO}}{dM^2 dx_F} = & \frac{4\pi\alpha_{em}^2}{3N_c M^4} \frac{\alpha_s(M^2)}{2\pi} \int_{z_{min}}^1 dz \frac{x_1 x_2}{x_1 + x_2} \sum_f e_f^2 \{q_f(x_1, M^2) \bar{q}_f(x_2, M^2) D_q(z) \\ & + g(x_1, M^2) [q_f(x_2, M^2) + \bar{q}_f(x_2, M^2)] D_g(z) + (x_1 \leftrightarrow x_2)\}, \end{aligned} \quad (6)$$

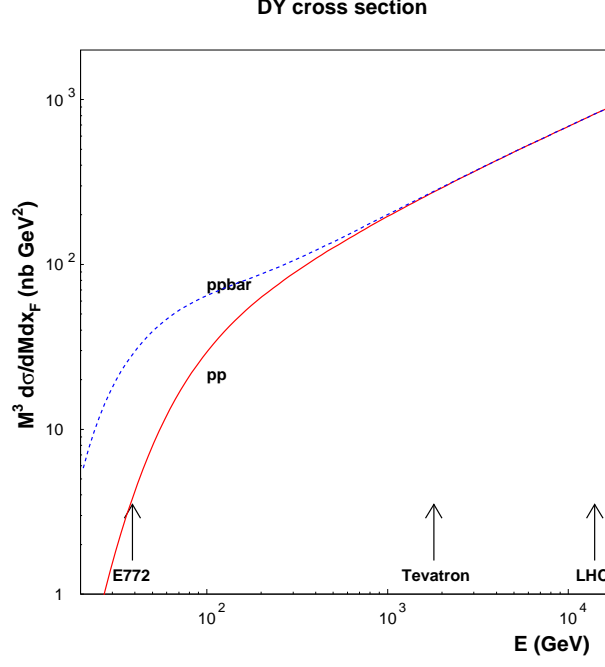


FIG. 2: The DY cross section in the collinear approach with CTEQ6.6M parton distributions as a function of center-of-mass energy $E = \sqrt{s}$ at fixed $x_F = 0.15$ and $M = 8$ GeV for pp and $p\bar{p}$ scattering.

where $x_{1,2}$ are given by relations (5) and g is a gluon distribution. The coefficient functions $D_{q,g}$ are of the form

$$D_q(z) = C_F \left\{ 4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2 \frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2\pi^2}{3} - 8 \right) \right\}, \quad (7)$$

$$D_g(z) = T_R \left\{ (z^2 + (1-z)^2) \ln \frac{(1-z)^2}{z} + \frac{1}{2} + 3z - \frac{7}{2}z^2 \right\}, \quad (8)$$

with $C_F = 4/3$, $T_R = 1/2$ and the standard (+) prescription to regularize soft gluon emission singularity at $z = 1$. Thus, the DY cross section in the collinear approximation up to $\mathcal{O}(\alpha_s)$ is the sum

$$\frac{d^2\sigma^{col}}{dM^2 dx_F} = \frac{d^2\sigma^{LO}}{dM^2 dx_F} + \frac{d^2\sigma^{NLO}}{dM^2 dx_F}. \quad (9)$$

The higher order (NNLO) corrections [8–10], proportional to α_s^2 , lead to much more complicated formulae for the coefficient functions and we will not provide them here.

III. pp VERSUS $p\bar{p}$ SCATTERING

We wish to compare the collinear factorization DY cross sections at the two presently operating colliders, the Femilab Tevatron which scatters $p\bar{p}$ and the CERN LHC which collides pp . Going from the proton into to the antiproton beam we have to interchange the quark distributions: $u \leftrightarrow \bar{u}$, $d \leftrightarrow \bar{d}$ etc..

Such a replacement will not affect the singlet quark distribution in the second term in eq. (7) but it will modify the terms with the product of the quark distributions. For example, for two flavours, in the LO approximation

$$d\sigma_{pp}^{LO} \sim e_u^2 \{u(x_1)\bar{u}(x_2) + \bar{u}(x_1)u(x_2)\} + e_d^2 \{d(x_1)\bar{d}(x_2) + \bar{d}(x_1)d(x_2)\}, \quad (10)$$

but in the $p\bar{p}$ case we find

$$d\sigma_{p\bar{p}}^{LO} \sim e_u^2 \{u(x_1)u(x_2) + \bar{u}(x_1)\bar{u}(x_2)\} + e_d^2 \{d(x_1)d(x_2) + \bar{d}(x_1)\bar{d}(x_2)\}. \quad (11)$$

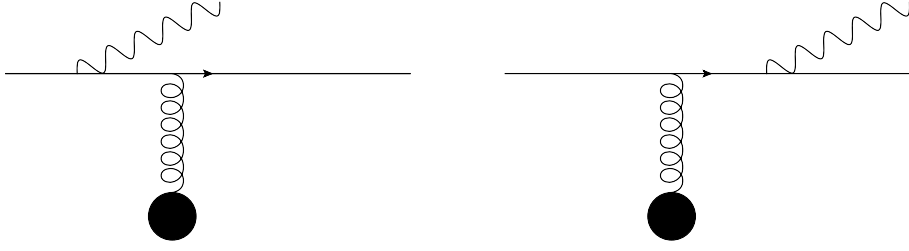


FIG. 3: The enhanced diagrams from Fig. 1 in the proton rest frame.

Both cross sections coincide if the momentum fraction x_2 is small. In such a case, to a good approximation, a sea quark is involved in the scattering since

$$u(x_2) \simeq \bar{u}(x_2), \quad d(x_2) \simeq \bar{d}(x_2). \quad (12)$$

This is illustrated in Fig. 2 where we show the DY cross section for pp and $p\bar{p}$ scattering as a function of the center-of-mass energy of colliding particles E at fixed $x_F = 0.15$ and $M = 8$ GeV. For $M \ll E$, $x_1 \approx x_F \sim 1$ and $x_2 \approx \tau/x_F \ll 1$, thus the two cross sections coincide. We use the NLO CTEQ6.6M parton distribution functions [20] for this comparison.

IV. SMALL x LIMIT

The small x or high energy limit of the DY process means that dilepton mass is much smaller than the center-of-mass energy of colliding particles, $M \ll \sqrt{s}$. When $x_1 \sim 1$ we have

$$x_2 = \frac{M^2}{s x_1} \ll 1, \quad (13)$$

i.e. in the parton model, a fast quark (or antiquark) with the momentum fraction x_1 annihilates with a slow antiquark (or quark) with the momentum fraction x_2 . In such a case, the diagrams in Figure 1c-1d, with a slow gluon, are particularly enhanced due to the strongly rising gluon distribution in the small- x limit. Now, a difficult task of small- x resummation arises [21], which touches the problem of unitarity corrections to the standard, linear QCD evolution equations.

One can reformulate this problem in the rest frame of one of the protons, which acts as a target. In this frame, the diagrams in Figure 1c-1d can be interpreted as the lowest order description of the process in which the fast quark scatters off a soft color field of the target with emission of a massive photon before or after the scattering, see Fig. 3. The photon subsequently decays into a pair of leptons.

The cross section for radiation of a virtual photon from the fast quark of flavour f , which takes a fraction z of the radiating quark energy, is given by [12]

$$\sigma_{T,L}^f(qp \rightarrow \gamma^* X) = \int d^2r W_{T,L}^f(z, r, M^2, m_f) \sigma_{qq}(x_2, zr), \quad (14)$$

where T, L denotes virtual photon polarisation, transverse or longitudinal, respectively. Here r is the photon-quark transverse separation and $W_{T,L}^f$ are equal to [11, 14]

$$W_T^f = \frac{\alpha_{em}}{\pi^2} \{ [1 + (1-z)^2] \eta^2 K_1^2(\eta r) + m_f^2 z^4 K_0^2(\eta r) \}, \quad (15)$$

$$W_L^f = \frac{2\alpha_{em}}{\pi^2} M^2 (1-z)^2 K_0^2(\eta r), \quad (16)$$

where $K_{0,1}$ are Bessel-McDonald functions, m_f is quark mass and $\eta^2 = (1-z)M^2 + z^2 m_f^2$.

The quantity σ_{qq} in eq. (14) is a dipole cross section known from DIS scattering at small Bjorken- x [22]. It was determined from fits to HERA data on the proton structure function F_2 at small x under different assumptions, e.g. assuming phenomenological form with a saturation scale $Q_s^2(x) = (x/x_0)^{-\lambda}$ [18, 23]:

$$\sigma_{qq}(x, r) = \sigma_0 \{ 1 - \exp(-r^2 Q_s^2(x)/4) \}. \quad (17)$$

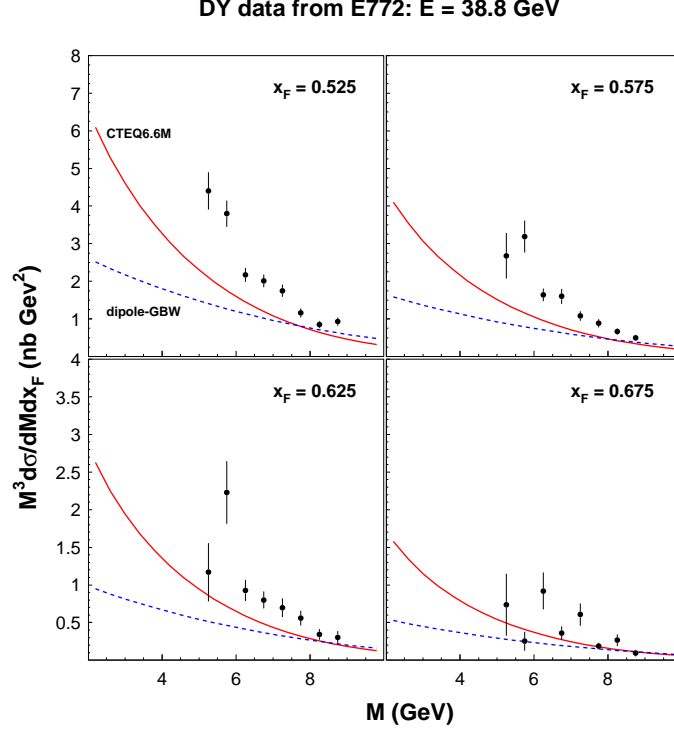


FIG. 4: The DY cross section from the collinear (solid lines) and dipole (dashed lines) approaches as a function of the dilepton mass M against the E772 collaboration data. The CTEQ6.6M parton distributions are used in the collinear case.

Substituting this dipole cross section into eq. (14) one can test predictions on the DY cross section in which parton saturation effects are taken into account. The final form of the DY cross section for the forward dilepton production is found after taking into account the incoming quark distribution in the proton

$$\frac{d^2\sigma_{T,L}^{DY}}{dM^2 dx_F} = \frac{\alpha_{em}}{6\pi M^2} \frac{x_1}{x_1 + x_2} \sum_f e_f^2 \int_{x_1}^1 \frac{dz}{z^2} \left[q_f\left(\frac{x_1}{z}, M^2\right) + \bar{q}_f\left(\frac{x_1}{z}, M^2\right) \right] \sigma_{T,L}^f(qp \rightarrow \gamma^* X). \quad (18)$$

The expression in the squared brackets under the integral is proportional to the LO contribution of flavour f to the F_2 proton structure function, $F_2^f = e_f^2 x(q_f + \bar{q}_f)$. Thus, the final formula reads

$$\frac{d^2\sigma_{T,L}^{DY}}{dM^2 dx_F} = \frac{\alpha_{em}}{6\pi M^2} \frac{1}{x_1 + x_2} \sum_f \int_{x_1}^1 \frac{dz}{z} F_2^f\left(\frac{x_1}{z}, M^2\right) \sigma_{T,L}^f(qp \rightarrow \gamma^* X). \quad (19)$$

A similar expression was found in [11] by changing the variables r and z to $\rho = zr$ and $\alpha = (1 - z)/z$. The new variable ρ has the interpretation of a size of a quark pair while α is a quark/antiquark longitudinal momentum fraction with respect to the photon momentum.

V. PREDICTIONS FOR THE LHC

In Figure 4 we present a comparison of the results from the collinear factorization formula (1) and the dipole formula (19) against the data from the Fermilab E772 collaboration [24]. We use the next-to-leading order (NLO) CTEQ6.6M parton distributions [20] for the collinear formula (solid lines) while the GBW parameterisation [25] of the dipole cross section was used in the dipole formula. In eq. (19), there are three massless light quarks and charm quark with mass $m_c = 1.4$ GeV. For the energy $\sqrt{s} = 38.8$ GeV and the indicated values of M and x_F , the fraction of the slow parton momentum $x_2 \approx 0.01 - 0.1$, which is slightly beyond the applicability of the dipole formula. We observe that the data are above the results from both the dipole and collinear factorization approaches. A similar result was found for the NLO MSTW08 parton distributions [26].

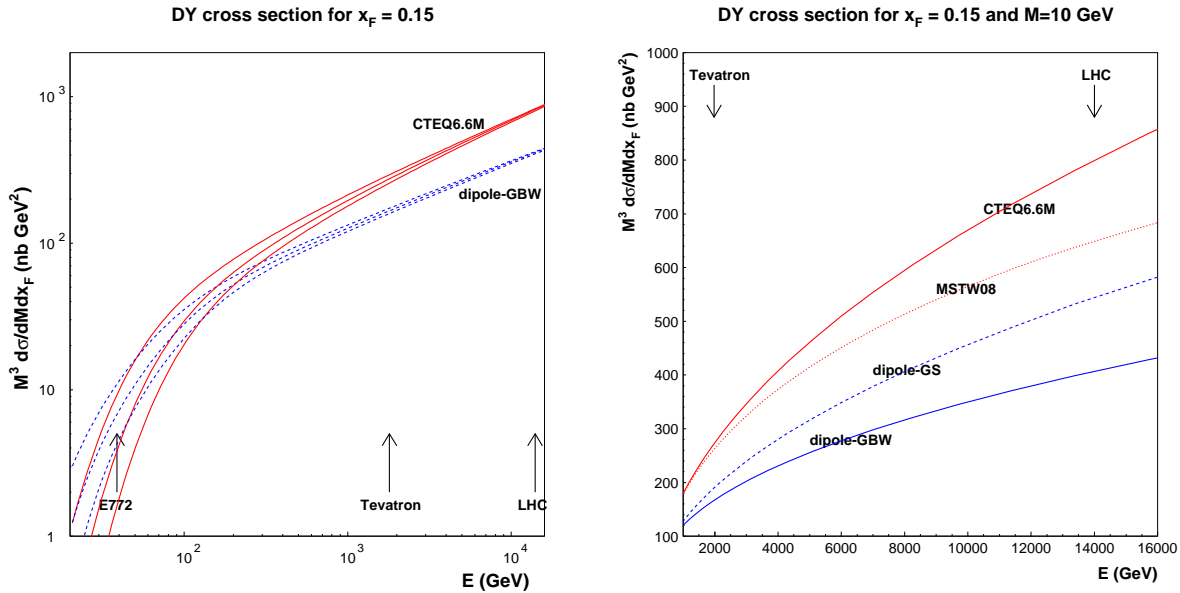


FIG. 5: Left: the DY cross section from the collinear (solid lines) and dipole (dashed lines) approaches as a function of energy $E = \sqrt{s}$ for fixed $x_F = 0.15$ and dilepton mass $M = 6, 8, 10$ GeV (from top to bottom). Right: the same for $M = 10$ GeV and the indicated parameterizations of the parton distributions and dipole cross section.

In Figure 5 (left) we present predictions for the DY cross section as a function of the center-of-mass energy $E = \sqrt{s}$ at fixed $x_F = 0.15$ and dilepton mass $M = 6, 8, 10$ GeV. For the collinear factorisation results we use the CTEQ6.6M parton distributions. In the figure on the right, we show the same results for $M = 10$ GeV in a more detailed way, using the linear scales. We additionally show the collinear factorization results for the MSTW08 parton distributions and the dipole approach results with two parameterizations of the dipole cross section: GBW [18] and GS [25]. In the latter parameterization the DGLAP evolution of the dipole cross section for small dipole sizes is built in. We also analysed the Color Glass Condensate parametrization [27], finding results very close to the GBW curves. At the LHC energy, the fraction $x_2 \approx 3 \cdot 10^{-6}$ and we are really in the small- x domain which has not been explored experimentally yet at the hard scale given by the invariant mass of the DY lepton pair. Thus, the presented results are only extrapolations. Nevertheless, we clearly see that saturation effects encoded in the dipole approach give results which are systematically below the collinear factorization predictions. At the LHC energy, in the most extreme case, the suppression of the DY cross section due to saturation effects can be as large as a factor of three.

We are looking forward to the experimental verification of this result.

VI. TWIST EXPANSION OF THE DIPOLE FORMULA

We will analyze the dipole DY cross section (19) from the point of view of the twist expansion in positive powers of the ratio $Q_s^2(x_2)/M^2$, where Q_s is the x -dependent saturation scale. We utilize the Mellin transform, using the methods elaborated in [17, 19]. The twist analysis for the Drell-Yan production is slightly more complicated than in the case of the structure functions. This is due to the fact that the integral in z cannot be performed analytically. This is because it involves the structure function $F_2(x_1/z)$ which is given as a parametrization to the experimental data. In order to compute the twist expansion we will present two methods. In the first one we will expand the integrand around the $z = 1$ point and perform the integrals analytically. This will give us approximate, analytical expressions for higher twists. In the second method we leave the integrals in z and devise a method to extract the twist contributions numerically. This method gives exact results for the twist expansion.

The standard definition of the Mellin transform of a function $f(r^2)$ reads

$$\phi(\gamma) = \int_0^\infty \frac{dr^2}{r^2} (r^2)^{-\gamma} f(r^2), \quad (20)$$

while the inverse transform is given by

$$f(r^2) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} (r^2)^\gamma \phi(\gamma), \quad (21)$$

where c is a real number which has to be taken in interval (a, b) such that the integral (20) is absolutely convergent for $a < \mathcal{R}e \gamma < b$.

We start the twist analysis from the transverse part of the cross section, and we assume that quarks are massless. The part of the DY cross section which corresponds to the transverse polarization of the photon (19) reads explicitly

$$\frac{d^2 \sigma_T^{DY}}{dM^2 dx_F} = \frac{\alpha_{em}^2}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \int_{x_1}^1 \frac{dz}{z} F_2\left(\frac{x_1}{z}, M^2\right) \int_0^\infty dr^2 [1 + (1-z)^2] M^2 (1-z) K_1^2(Mr\sqrt{1-z}) \sigma_{q\bar{q}}(x_2, zr). \quad (22)$$

Using the above definitions of the Mellin transform, the dipole cross section can be written as

$$\sigma_{q\bar{q}}(x, r) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} (r^2)^\gamma \int_0^\infty \frac{dr'^2}{r'^2} (r'^2)^{-\gamma} \sigma_{q\bar{q}}(x, r'). \quad (23)$$

We will perform the twist analysis using the GBW parametrization of the dipole cross section (17) which is given in a closed analytic form. The advantage of this is that we can perform the integral over the dipole sizes analytically.

The Mellin transform of the GBW cross section reads

$$\sigma_0 G(\gamma) \equiv \sigma_0 \int_0^\infty \frac{d\hat{r}^2}{\hat{r}^2} (\hat{r}^2)^{-\gamma} (1 - e^{-\hat{r}^2}) = -\sigma_0 \Gamma(-\gamma), \quad (24)$$

and therefore has single poles for all non-negative integer values of γ .

Using representation (23) together with the explicit Mellin transform of the GBW cross section (24), we can rewrite the above expression as

$$\begin{aligned} \frac{d^2 \sigma_T^{DY}}{dM^2 dx_F} &= \frac{\alpha_{em}^2}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \int_{x_1}^1 \frac{dz}{z} F_2\left(\frac{x_1}{z}, M^2\right) \int_0^\infty dr^2 [1 + (1-z)^2] M^2 (1-z) K_1^2(Mr\sqrt{1-z}) \\ &\times \sigma_0 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} (r^2)^\gamma \left(\frac{z^2 Q_s^2(x_2)}{4}\right)^\gamma G(\gamma), \end{aligned} \quad (25)$$

where the contour of integration over γ is chosen so that the constant c satisfies $0 < \mathcal{R}e c < 1$ and $\mathcal{I}m c = 0$. We can now perform the integration over the dipole size r . To this aim we define

$$\tilde{H}_T(\gamma) \equiv \int_0^\infty d\tilde{r}^2 K_1^2(\tilde{r}) (\tilde{r}^2)^\gamma = \frac{\sqrt{\pi} \Gamma(\gamma) \Gamma(1+\gamma) \Gamma(2+\gamma)}{2\Gamma(\frac{3}{2} + \gamma)}. \quad (26)$$

The transverse part of the DY cross section reads therefore

$$\begin{aligned} \frac{d^2 \sigma_T^{DY}}{dM^2 dx_F} &= \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} G(\gamma) \tilde{H}_T(\gamma) \left(\frac{Q_s^2(x_2)}{4M^2}\right)^\gamma \\ &\times \int_{x_1}^1 \frac{dz}{z} F_2\left(\frac{x_1}{z}, M^2\right) [1 + (1-z)^2] \left(\frac{z^2}{1-z}\right)^\gamma. \end{aligned} \quad (27)$$

Clearly the poles in the γ plane control the behavior in M^2 . We need to evaluate the integral over the longitudinal momentum fraction z . Let us introduce the following notation

$$I_{T,\gamma}(x_1, z, M^2) = \frac{1}{z} F_2\left(\frac{x_1}{z}, M^2\right) [1 + (1-z)^2] (z^2)^\gamma. \quad (28)$$

We first perform the integral over z analytically by expanding the above expression around $z = 1$. We assume that $F_2(x_1/z, M^2)$ does not have any singularities at $z = 1$, which is corroborated by the expressions for the structure functions at these values of $x_1 \sim 1$.

Expanding the integrand and integrating term by term one obtains,

$$\int_{x_1}^1 dz \sum_{k \geq 0} I_{T,\gamma}^{(k)}(x_1, z = 1, M^2) (1-z)^k \left(\frac{1}{1-z}\right)^\gamma = \sum_{k \geq 0} I_{T,\gamma}^{(k)}(x_1, z = 1, M^2) (1-x_1)^{1-\gamma+k} \frac{1}{1-\gamma+k}. \quad (29)$$

Inserting this expansion into eq. (27) one obtains a general expression which allows to extract the twists systematically in powers of $(1 - x_1)$

$$\frac{d^2 \sigma_T^{DY}}{dM^2 dx_F} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \sum_{k \geq 0} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} G(\gamma) \tilde{H}_T(\gamma) \frac{I_{T,\gamma}^{(k)}(x_1, z=1, M^2)}{1 - \gamma + k} (1 - x_1)^{1+k} \left(\frac{Q_s^2(x_2)}{4M^2(1 - x_1)} \right)^\gamma. \quad (30)$$

The twist expansion then corresponds to taking residues of the different poles in γ that appear on the right-hand side of (30). One can also rewrite it to expose a more general structure of this expansion

$$\frac{d^2 \sigma_T^{DY}}{dM^2 dx_F} = \sum_{\gamma_c \geq 1} \sum_{k \geq 1 - \gamma_c} (1 - x_1)^k \left(\frac{Q_s^2(x_2)}{4M^2} \right)^{\gamma_c} C_{k, \gamma_c}(x_1, \ln Q_s^2/(M^2(1 - x_1)), \ln M^2), \quad (31)$$

where the first sum is performed over the poles in eq. (30). The coefficients C_{k, γ_c} depend on $\ln Q_s^2/(M^2(1 - x_1))$ which reflects the fact that there can be multiple poles, and the $\ln M^2$ dependence comes from the possible dependence of F_2 on M^2 . Since F_2 is evaluated at large values of x_1 we expect this dependence to be very mild.

A. Twist 2

To extract the twist 2 in powers of $Q_s^2(x_2)/M^2$ we will consider the leading term in the expansion of the function I_T around $z = 1$. Taking this term, the integral over z reads

$$\begin{aligned} \int_{x_1}^1 dz I_{T,\gamma}^{(0)}(x_1, z=1, M^2) \left(\frac{1}{1-z} \right)^\gamma &= \int_{x_1}^1 dz F_2(x_1, M^2) \left(\frac{1}{1-z} \right)^\gamma \\ &= F_2(x_1, M^2) (1 - x_1)^{1-\gamma} \frac{1}{1-\gamma}. \end{aligned} \quad (32)$$

Therefore integral over z gives the single pole in the γ plane. The function $G(\gamma) = -\Gamma(-\gamma)$ also has a single pole, which together gives the double pole in $\gamma = 1$. The higher order terms in expansion of (28) will contribute to terms which are suppressed by powers of $(1 - x_1)$ as is evident from eq. (31). We will discuss these later.

Taking the first term in the expansion in $1 - z$, (32) we obtain the approximate formula

$$\frac{d^2 \sigma_T^{DY}}{dM^2 dx_F} \simeq \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{F_2(x_1, M^2)}{x_1 + x_2} (1 - x_1) \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} G(\gamma) \tilde{H}_T(\gamma) \frac{1}{1-\gamma} \left(\frac{Q_s^2(x_2)}{4M^2(1 - x_1)} \right)^\gamma. \quad (33)$$

The leading twist extraction amounts to taking the above formula and closing the contour to the right and taking the contribution from the pole at $\gamma = 1$. The approximate expression for the leading twist is therefore

$$\Delta_{T,2}^{(0)} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{F_2(x_1, M^2)}{x_1 + x_2} \times 2 \frac{Q_s^2(x_2)}{4M^2} \left[\frac{4}{3} \gamma_E - 1 + \frac{2}{3} \psi\left(\frac{5}{2}\right) - \frac{2}{3} \ln \frac{Q_s^2(x_2)}{4M^2(1 - x_1)} \right] + \mathcal{O}(1 - x_1). \quad (34)$$

Note that there is an additional dependence on the mass M^2 in the function $F_2(x_1, M^2)$. As $x_1 \simeq 1$ this dependence should be very mild and should not affect too much the twist expansion we are performing. We stress that this expression is an approximate one in the sense that there are subleading contributions coming from the other terms in expression (28). They will not modify however the logarithmic term as they are regular as $\gamma = 1$. More specifically the first subleading contribution to the leading twist comes from

$$\begin{aligned} \int_{x_1}^1 dz I_{T,\gamma}^{(1)}(x_1, z=1, M^2) \left(\frac{1}{1-z} \right)^\gamma &= \int_{x_1}^1 dz (F_2(x_1, M^2) (1 - 2\gamma) + x_1 F_2'(x_1, M^2)) \left(\frac{1}{1-z} \right)^{\gamma-1} \\ &= [F_2(x_1, M^2) (1 - 2\gamma) + x_1 F_2'(x_1, M^2)] (1 - x_1)^{2-\gamma} \frac{1}{2-\gamma}. \end{aligned} \quad (35)$$

This contribution is the leading one for the twist 4, but it also will affect the twist 2 as it multiplies the single pole in $\gamma = 1$ present in $G(\gamma)$. It gives the following correction to the leading expression for twist 2, eq. (34):

$$\Delta_{T,2}^{(1)} = -\frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{[F_2(x_1, M^2) - x_1 F_2'(x_1, M^2)]}{x_1 + x_2} (1 - x_1) \frac{Q_s^2(x_2)}{4M^2} \times \frac{4}{3}, \quad (36)$$

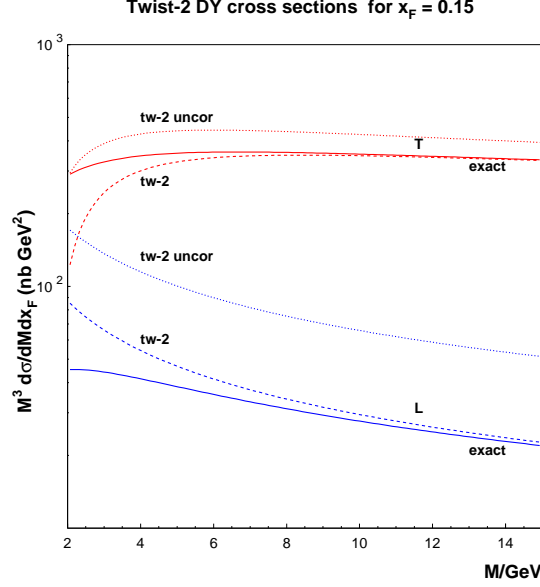


FIG. 6: The twist-2 contributions, shown by the dashed lines, for the transverse and longitudinal DY cross section at the LHC energy $\sqrt{s} = 14$ TeV. The solid lines show the all-twist results while the dotted lines correspond to the approximate twist-2 results given by eqs. (34) and (A6).

which is suppressed by one power of $(1 - x_1)$.

The above-presented method allows to systematically compute the contributions to the given twist in powers of $(1 - x_1)$. The effectiveness of this calculation depends however on the rate of the convergence of the series in $(1 - x_1)$. In our kinematics x_1 is about $0.1 - 0.2$, hence this convergence is rather slow. Below we will present another semi-analytical method which allows to compute the leading twist expression exactly.

As the double pole contribution has been evaluated already in (34), the remaining single pole contribution comes from the pole in $G(\gamma)$ multiplying the non-singular in $\gamma = 1$ part of the integral over z . It suffices to take the regularized integral

$$\int_{x_1}^1 dz \frac{I_{T,\gamma=1}(x_1, z, M^2) - I_{T,\gamma=1}^{(0)}(x_1, z=1, M^2)}{1 - z} = \int_{x_1}^1 dz \frac{z F_2(\frac{x_1}{z}, M^2)(1 + (1 - z)^2) - F_2(x_1, M^2)}{1 - z}, \quad (37)$$

which is evaluated at $\gamma = 1$. The integrand in this expression is regular at $z = 1$ and thus the integral can be evaluated purely numerically. The correction to eq. (34) is therefore

$$\Delta_{T,2}^{(k>0)} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \times \frac{4}{3} \left(\frac{Q_s^2(x_2)}{4M^2} \right) \int_{x_1}^1 dz \frac{z F_2(\frac{x_1}{z}, M^2)(1 + (1 - z)^2) - F_2(x_1, M^2)}{1 - z}, \quad (38)$$

and the complete, exact leading twist expression equals

$$\frac{d^2 \sigma_T^{DY(\tau=2)}}{dM^2 dx_F} = \Delta_{T,2}^{(0)} + \Delta_{T,2}^{(k>0)}. \quad (39)$$

A similar twist-2 analysis of the longitudinal part of the DY cross section is given in Appendix A.

In Fig. 6 we show the comparison of the calculation of the all-twist formulae, eq. (27) for the transverse and eq. (A2) for the longitudinal parts (shown by the solid lines), with the approximate leading twist contribution. The approximate twist-2 contributions $\Delta_{T,2}^{(0)}$ and $\Delta_{L,2}^{(0)}$, eqs. (34) and (A6) respectively, are shown by the dotted lines while the exact twist-2 expressions, eq. (39) and eq. (A7), are depicted by the dashed lines. We see that the approximate twist-2 formulae are significantly above the exact result even for larger values of M . This is due to the large value of the subleading terms in the expansion of $(1 - x_1)$. Since $x_1 \simeq x_F = 0.15$ the higher order terms can be still contributing. On the other hand, we observe that the exact twist-2 formulae are getting close to the all-twist result in the region $M > 6$.

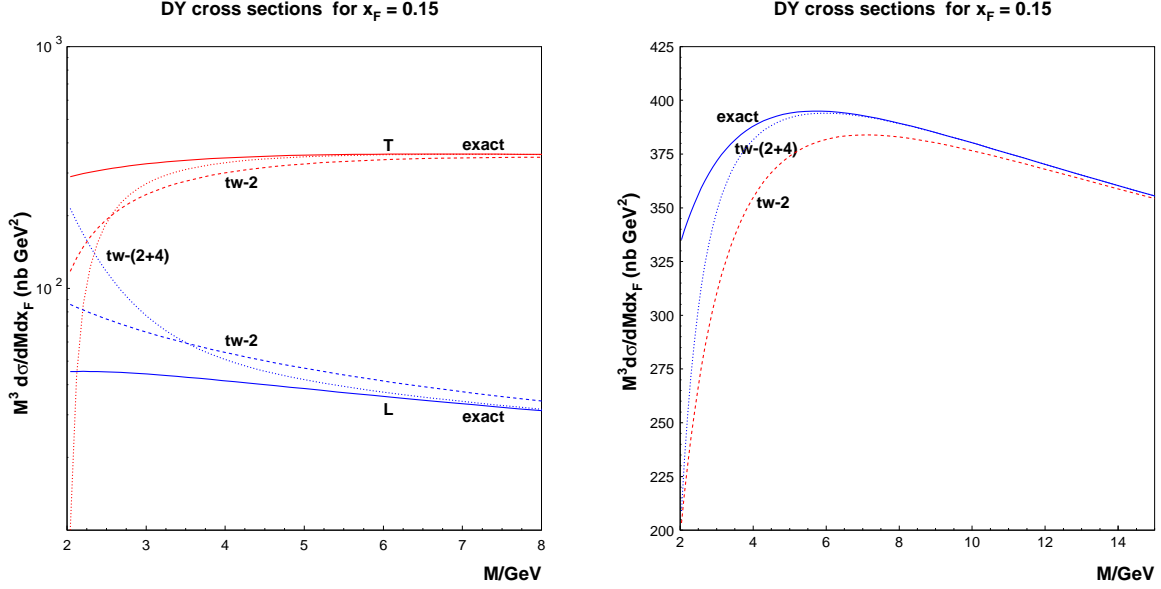


FIG. 7: Convergence of the twist expansion of the transverse and longitudinal DY cross sections (left) and of their sum (right). The solid lines show the all-twist results while the dashed and dotted lines correspond to the twist-2 and twist-(2+4) contributions, respectively.

B. Twist 4

In this section we provide semi-analytical expressions for the twist 4 both for the longitudinal and transverse polarizations of the DY photon. We start with the transverse part and use eqs. (27) and (35) together, which will give part of the twist 4 contribution. Taking the residue at $\gamma = 2$ we obtain

$$\Delta_{T,4}^{(1)} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \left(\frac{Q_s^2(x_2)}{4M^2} \right)^2 \times \frac{4}{15} \left[F_2(x_1, M^2) \left(-63 + 36\gamma_E + 18\psi(7/2) - 18 \log \left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)} \right) \right) \right. \\ \left. + x_1 F_2'(x_1, M^2) \left(17 - 12\gamma_E - 6\psi(7/2) + 6 \log \left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)} \right) \right) \right]. \quad (40)$$

The second part contributing to twist four comes from the contribution of the single pole in $G(\gamma)$ multiplying the finite part in the integral over z . This can be evaluated in the similar manner as before by modifying the integral in order to perform analytical continuation to $\gamma = 2$. To this aim we subtract in the integrand the first two terms of the expansion in $(1-z)$

$$\delta\mathcal{F}(x_1, M^2) = \int_{x_1}^1 dz \frac{z^3 F_2(\frac{x_1}{z}, M^2) (1 + (1-z)^2) - F_2(x_1, M^2) - (1-z)[-3F_2(x_1, M^2) + x_1 F_2'(x_1, M^2)]}{(1-z)^2}, \quad (41)$$

which gives convergent integral by definition. Then we need to add the term coming from the lowest order term in expansion in $z = 1$ evaluated at $\gamma = 2$ which is

$$-\frac{F_2(x_1, M^2)}{1-x_1}.$$

The expression contributing to twist 4 is thus

$$\Delta_{T,4}^{(2)} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{1}{x_1 + x_2} \left(\frac{Q_s^2(x_2)}{4M^2} \right)^2 \times \left(-\frac{8}{5} \right) \left[\delta\mathcal{F}(x_1, M^2) - \frac{F_2(x_1, M^2)}{1-x_1} \right], \quad (42)$$

where we evaluated the residue coming from the single pole at $\gamma = 2$. The final expression for twist 4 for the transverse part is therefore

$$\frac{d^2 \sigma_T^{DY(\tau=4)}}{dM^2 dx_F} = \Delta_{T,4}^{(1)} + \Delta_{T,4}^{(2)}. \quad (43)$$

In Fig. 7 (left) we show convergence of the twist expansion for the transverse and longitudinal DY cross sections (left plot) and their sum (right plot). The solid lines show the all-twist results while the dashed and dotted lines correspond to the twist-2 and twist-(2+4) contributions, respectively. We see that there is a poor convergence below $M < 6$ GeV. Both twist contributions are below the exact results for the transverse part of the cross section while for the longitudinal part they are above the exact result. This is the reason why the sum of the transverse and longitudinal parts of the twist contributions approaches the exact total cross section. This effect is shown in Fig. 7 (right).

Summary

In this paper we have investigated Drell-Yan production at forward rapidities and at high energies accessible at the LHC. We have used the dipole formulation suitable for forward rapidities and used the cross section which incorporates saturation effects. The comparison with the standard collinear formula shows the suppression of the production cross section for highest energies when using the dipole models with saturation effects.

Using the dipole formulation with the GBW formula for the dipole model cross section, we have constructed the twist expansion for this process. Unlike the DIS case, where the twist expansion for the GBW model could be performed completely analytically, here we had to resort to the semi-analytical evaluation of the individual terms in the expansion. It was shown that the leading twist is a good approximation to the full result for masses of the Drell-Yan pair larger than about ~ 6 GeV. For lower masses the twist expansion quickly becomes divergent and full resummation is necessary.

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Appendix A: Twist decomposition of the longitudinal part

The leading twist-2 contribution for the longitudinal part of the cross section can be computed in a similar manner as for the transverse part. We use

$$\tilde{H}_L(\gamma) \equiv \int_0^\infty d\tilde{r}^2 K_0^2(\tilde{r})(\tilde{r}^2)^\gamma = \frac{\sqrt{\pi} \Gamma(1+\gamma)^3}{2\Gamma(\frac{3}{2}+\gamma)}. \quad (A1)$$

After performing the integrals over the transverse coordinate and using the Mellin representation we obtain

$$\frac{d^2 \sigma_L^{DY}}{dM^2 dx_F} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{2}{x_1 + x_2} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} G(\gamma) \tilde{H}_L(\gamma) \left(\frac{Q_s^2(x_2)}{4M^2} \right)^\gamma \int_{x_1}^1 \frac{dz}{z} F_2\left(\frac{x_1}{z}, M^2\right) (1-z) \left(\frac{z^2}{1-z} \right)^\gamma. \quad (A2)$$

Similarly to the transverse case, we can introduce the function

$$I_L(x_1, z, M^2) = \frac{1}{z} F_2\left(\frac{x_1}{z}, M^2\right) (z^2)^\gamma. \quad (A3)$$

The leading term coming from the expansion around $z = 1$ will thus give

$$\int_{x_1}^1 dz I_L^{(0)}(x_1, z = 1, M^2) \left(\frac{1}{1-z} \right)^{\gamma-1} = \int_{x_1}^1 dz F_2(x_1, M^2) \left(\frac{1}{1-z} \right)^{\gamma-1} = F_2(x_1, M^2) (1-x_1)^{2-\gamma} \frac{1}{2-\gamma}. \quad (A4)$$

Note that this expression is not singular at leading pole $\gamma = 1$. Again the leading twist extraction in the longitudinal case therefore amounts to taking the formula

$$\frac{d^2\sigma_L^{DY}}{dM^2 dx_F} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{2F_2(x_1, M^2)}{x_1 + x_2} (1 - x_1)^2 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} G(\gamma) \tilde{H}_L(\gamma) \frac{1}{2-\gamma} \left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)} \right)^\gamma, \quad (\text{A5})$$

and closing the contour to the right, enclosing the pole at $\gamma = 1$. The final result is

$$\Delta_{L,2}^{(0)} \simeq \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{2F_2(x_1, M^2)}{x_1 + x_2} (1 - x_1) \frac{Q_s^2(x_2)}{4M^2} \times \frac{2}{3} + \mathcal{O}((1-x_1)^2). \quad (\text{A6})$$

There are no logarithmic corrections because of the single leading pole coming only from the function $G(\gamma)$.

Using analogous method as before we can also evaluate the exact expression for the longitudinal part of the cross section. The integral over z in (A2) is well defined for $\gamma = 1$ and so to evaluate the exact twist 2 we can perform this integral directly numerically. The exact expression for the twist 2 contribution to the longitudinal part of the cross section reads

$$\frac{d^2\sigma_L^{DY(\tau=2)}}{dM^2 dx_F} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{2}{x_1 + x_2} \frac{Q_s^2(x_2)}{4M^2} \times \frac{2}{3} \int_{x_1}^1 dz z F_2\left(\frac{x_1}{z}, M^2\right). \quad (\text{A7})$$

Analogously we evaluate the twist-4 contribution to the longitudinal part. The first part of this contribution is obtained by taking the residue at $\gamma = 2$ of expression (A5)

$$\Delta_{L,4}^{(1)} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{2F_2(x_1, M^2)}{x_1 + x_2} \left(\frac{Q_s^2(x_2)}{4M^2} \right)^2 \times \left(-\frac{16}{15} \right) \left[-3 + 2\gamma_E - \log \left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)} \right) + \psi \left(\frac{7}{2} \right) \right]. \quad (\text{A8})$$

The second part comes again from the finite part of integral in z which is

$$\int_{x_1}^1 dz \frac{I_{L,\gamma=2}(x_1, z, M^2) - I_{L,\gamma=2}^{(0)}(x_1, z=1, M^2)}{1-z} = \int_{x_1}^1 dz \frac{z^3 F_2\left(\frac{x_1}{z}, M^2\right) - F_2(x_1, M^2)}{1-z}. \quad (\text{A9})$$

We thus obtain

$$\Delta_{L,4}^{(2)} = \frac{\alpha_{em}^2 \sigma_0}{6\pi^2 M^2} \frac{2}{x_1 + x_2} \left(\frac{Q_s^2(x_2)}{4M^2} \right)^2 \times \left(-\frac{16}{15} \right) \int_{x_1}^1 dz \frac{z^3 F_2\left(\frac{x_1}{z}, M^2\right) - F_2(x_1, M^2)}{1-z}. \quad (\text{A10})$$

The total contribution to the twist 4 for the longitudinal case is therefore

$$\frac{d^2\sigma_L^{DY(\tau=4)}}{dM^2 dx_F} = \Delta_{L,4}^{(1)} + \Delta_{L,4}^{(2)}. \quad (\text{A11})$$

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